Science Olympiad
UT Invitational
October 26, 2024

Astronomy C Walkthrough

In this walkthrough, we will go over Sections B (JS9), C (Deep-Sky Objects), and D (Astrophysics). References to various sources (e.g. online material and textbooks) are included to guide the reader towards resources to learn the concepts more in depth. We hope readers find it useful.

Section A: General Knowledge

This section consists of multiple choice questions about general astronomy concepts. Each question is worth 2 points, for a total of 60 points.

For the first five (5) questions, consider the H-R Diagram in Image 1.

1. The Sun is located at which point? **D**
2. Towards which point will the Sun evolve to after it leaves the main sequence? **C**
3. A star whose energy source is solely gravitational contraction would be at which point? **E**
4. A red giant would be at which point? **C**
5. A star at the center of a planetary nebula would be located at which point? **F**

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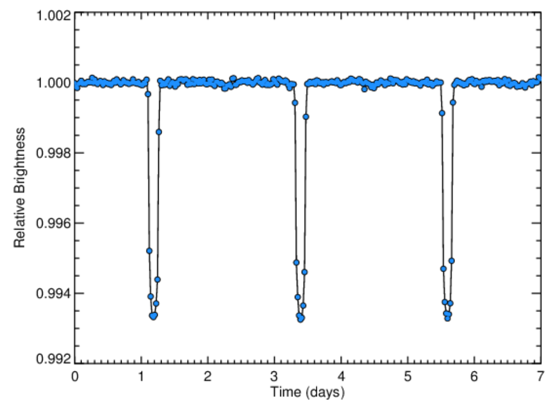
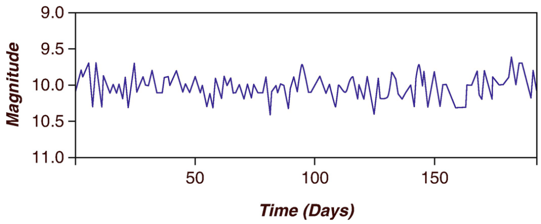
6. What type of scale is typically used for the axes of the H-R diagram?
   - A. Linear
   - B. Quadratic
   - C. Semi-logarithmic
   - **D. Logarithmic**
7. Which spectral class of stars has the highest surface temperature?
   - A. K
   - B. M
   - C. G
   - **D. O**
8. What is the characteristic feature of an H II region in terms of its spectral lines?
   - A. Continuum emission without lines
   - **B. Emission lines from ionized hydrogen**
   - C. Strong absorption lines
   - D. Emission of neutral hydrogen
9. What does a negative B-V color index signify about a star's temperature?
   - A. The star has an average temperature
   - **B. The star is relatively hot**
   - C. The star is pulsating
   - D. The star is relatively cool



10. What is typically responsible for periodic dips in the brightness of the light curve shown?
    - A. Sunspots
    - B. Intrinsic opacity fluctuations
    - **C. Planet orbiting the star**
    - D. Interstellar gas cloud
11. What feature distinguishes a brown dwarf from a true star?
    - A. Brown dwarfs emit strong X-rays
    - B. Brown dwarfs are always cooler than M-type stars
    - C. Brown dwarfs form from planetary nebulae
    - **D. Brown dwarfs cannot fuse hydrogen in their cores**
12. What spectral feature is commonly associated with molecular clouds?
    - **A. Rotational transitions of molecules like CO**
    - B. X-ray emission
    - C. Hα emission lines
    - D. Strong UV absorption



13. What type of star is the above image a light curve of?

    A. Cepheid

    B. Herbig Ae/Be

    C. RV Tauri

    D. T Tauri

14. Which phenomenon is associated with Herbig-Haro objects?

    A. Outflows from young stars

    B. Spiral arm formation in galaxies

    C. Supernova remnants

    D. Accretion onto a black hole

15. What is the primary difference between Herbig Ae/Be stars and T Tauri stars?

    A. T Tauri stars lack circumstellar disks

    B. T Tauri stars are older

    C. Herbig Ae/Be stars have strong magnetic fields

    D. Herbig Ae/Be stars are more massive

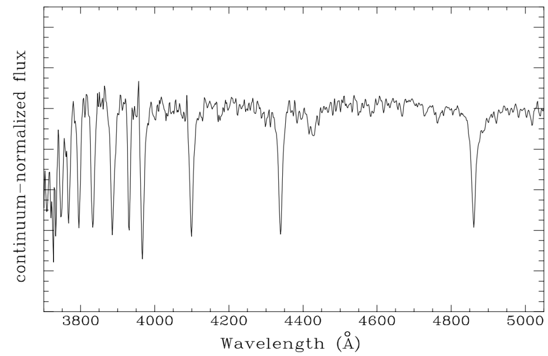
16. Which type of star would produce a spectral energy distribution curve with the shortest peak wavelength?

    A. A blue giant

    B. A red dwarf

    C. A solar-mass main sequence star

    D. A red giant



17. What type of star is most likely to produce the spectrum shown above?

    A. O

    B. K

    C. G

    D. A

18. What is the primary reason that Herbig Ae/Be stars exhibit strong emission lines in their spectra?

    A. Accretion of material from surrounding gas disks

    B. High-energy flares from magnetic fields

    C. Large-scale winds from their outer envelopes

    D. Shock waves from stellar pulsations

19. Why does the radius of a brown dwarf decrease as its mass increases beyond a certain point?

    A. Cooling from radiative losses as mass increases

    B. Increased radiative pressure from nuclear burning

    C. Onset of electron degeneracy pressure

    D. Collapse of the brown dwarf into a neutron star

20. Why do sub-Neptune planets often have thick atmospheres despite their smaller sizes compared to Jupiter?
    - A. Their cores are heated by tidal forces from nearby stars
    - B. Their lower masses allow them to retain hydrogen-rich atmospheres more easily
    - **C. Their formation in cooler regions of protoplanetary disks facilitates gas accretion**
    - D. Their atmospheres are maintained by strong magnetic fields

21. Why are hot Jupiters more easily detectable via radial velocity than Earth-like planets?
    - A. They are located at greater distances from their stars
    - B. They have higher temperatures, making them more luminous
    - **C. They induce stronger gravitational forces on their host stars**
    - D. They have larger radii and shorter orbital periods

22. What is the primary reason that super-Earths and sub-Neptunes are difficult to distinguish using only the transit method?
    - **A. Both types of planets produce similar transit depths**
    - B. Their masses are too small to create significant radial velocity signals
    - C. Their atmospheric compositions are indistinguishable
    - D. The orbital periods of these planets overlap significantly

23. How does the amplitude of the radial velocity signal depend on the orbital inclination of the planet relative to our line of sight? Note that an inclination of $0°$ is face-on and $90°$ is edge-on.
    - A. It increases to a maximum value as the inclination approaches $0°$
    - B. It increases to infinity as the inclination approaches $0°$
    - **C. It increases to a maximum value as the inclination approaches** $90°$
    - D. It increases to infinity as the inclination approaches $90°$

24. What type of star would make detecting an Earth-sized planet using the radial velocity method most difficult?
    - A. A moderately sized G-type star
    - **B. A large, massive O-type star**
    - C. A low-mass K-dwarf star
    - D. A small, dim M-dwarf star

25. How would a planet's atmospheric composition influence the depth of its transit as observed from Earth?
    - **A. Planets with hazy or high-opacity atmospheres show deeper transits at infrared wavelengths**
    - B. Planets with thick atmospheres would have shallower transits due to scattering effects
    - C. A planet with a thick atmosphere rich in hydrogen would have deeper transits at optical wavelengths
    - D. A planet's transit depth is unaffected by its atmospheric composition

26. Many hot gas giants have belts of wind around their equator which distribute heat from the warm dayside to the cool nightside of the planet. This process is most aptly characterized by what fundamental mode of heat transfer?
    A. Conduction
    B. Convection
    C. Radiation
    D. Advection

27. Image 2 shows a cluster of population I stars. Which of the following is true?
    A. It is a globular cluster and has a low rate of star formation.
    B. It is a globular cluster and has a high rate of star formation.
    C. It is an open cluster and has a low rate of star formation.
    D. It is an open cluster and has a high rate of star formation.

28. In a globular cluster with a half-mass radius of 10 pc, stars are interacting via gravitational encounters. What happens to the central density and velocity dispersion of the stars if the cluster undergoes core collapse?
    A. The central density remains constant, but velocity dispersion decreases
    B. The central density increases and velocity dispersion decreases
    C. The central density increases and velocity dispersion increases
    D. The central density decreases and velocity dispersion decreases

29. A molecular cloud is found within a large galaxy, and the local velocity dispersion within the cloud is measured to be 10 km/s. How might the galaxy affect the collapse of the clouds and the formation of stars?
    A. Tidal forces stretch the clouds, decreasing the Jeans mass and promoting low-mass star formation
    B. The external gravitational potential increases the velocity dispersion, leading to a higher Jeans mass and fewer low-mass stars forming
    C. The external gravitational potential enhances collapse by compressing the cloud, leading to rapid star formation
    D. Tidal forces from the galaxy cluster prevent cloud collapse, lowering the Jeans mass and inhibiting star formation

30. A planetary system forms around a Herbig Ae star, and observations show that the system contains both a sub-Neptune and a super-Earth. How would the planet migration patterns and disk clearing timescales differ between these planets?
    A. The super-Earth migrates more slowly due to Type I migration, while the sub-Neptune experiences Type II migration and clears a gap
    B. Both planets undergo rapid Type II migration, but the sub-Neptune clears a gap faster than the super-Earth
    C. The sub-Neptune migrates more rapidly due to its higher mass, while the super-Earth clears a gap more quickly
    D. The sub-Neptune undergoes slower Type II migration, while the super-Earth migrates rapidly due to lower mass

Section B: JS9

This section consists of a lab using the JS9 imaging software. Points are shown for each question, for a total of 18 points.

Setup Instructions

- Go to chandra.cfa.harvard.edu/js9
- Select the button on the right with the text `[The Unofficial Chandra Archive Search Page]`. A pop-up should appear.
- In the `[Object Name]` box, enter "`DS Tucanae`" and select `[Search]`.

1. [1 pt] How many results appear?

In the table of observations (you may need to scroll down in the pop-up window), find the row with `ObsID 25103` and the column labeled `Title`. To load the data, drag and drop the link there onto the JS9 window.

To adequately see the object, make sure that `[Scale > log]` is selected; you may also need to adjust the contrast and bias by holding down left click in the JS9 window and moving up/down and left/right, respectively.

At this point, you should be able to see the visual binary star. This is an image of DS Tucanae, a binary star system with a confirmed exoplanet!

2. [2 pts] What date did the observation start?
3. [2 pts] What instrument was used to create this image?
4. [3 pts] What is the angular separation between the two stars in arcseconds?
5. [3 pts] If DS Tucanae is 44.1 parsecs away from Earth, compute its separation in au. Show your work!

Set two circular regions and move one to encompass the primary and the other to encompass the secondary. It will be used for the next two questions.

6. [4 pts] On each star, run a light curve with `[Analysis > Light Curve]`. (Be sure to use the light curve routine listed under "`Server-side Analysis`".) Briefly (1-2 sentences) describe the results.
7. [3 pts] For each star, is there a noticeable change in their brightness, and if so, by what percent does it change? (Round to the nearest 10 %.)

Solution: You can find a video walkthrough of the JS9 lab here!

There are a ton of great material available for you to get comfortable with JS9. The main JS9 site has guided tutorials and past JS9 labs along with their solutions. Another useful resource is the community-made Scioly.org wiki page for JS9.

As mentioned in the walkthrough, question 5 uses the relationship between angular size, linear size, and distance. You can find a short introduction to how astronomers use angular size here.

Section C: Deep-Sky Objects

This section consists of short free-response questions about this year's deep-sky objects. Points are shown for each sub-question, for a total of 55 points.

1. The cover image shows a composite image of a region of star formation.
   (a) [1.5 pts] What is the name of this object?
   (b) [2 pts] This composite image displays two different wavelengths. Identify them.
   (c) [2 pts] Which telescope collected the longer wavelength data?
   (d) [2 pts] What types of objects, typically shrouded in dust, are revealed when observing in this wavelength?

Solution:

(a) 30 Doradus or Tarantula Nebula. This year, there are only two objects not related to exoplanets—the other one being the Orion Nebula. Be sure to get familiar with the rough structure/shape of the two nebulas, so even if you don't have the particular image saved, you would be able to make an educated guess.

(b) This image is from NASA and is a composite of X-ray, by the Chandra X-ray Observatory, and infrared, by the James Webb Space Telescope (JWST).

(Rabbit Hole: A large percentage of images you will encounter in Astronomy point back to NASA's Great Observatories, particularly of "classical" objects that have been well studied for decades. It's a fun deep dive to learn about the telescopes of the past, observatories active today, and up and coming ones.)

(c) JWST images in infrared, which has a longer wavelength than X-ray, which the Chandra telescope collects. Everything we know about the stars comes from the light we receive from them, so understanding the electromagnetic spectrum is critical, and leads us to multiwavelength astronomy!

(d) Protostars. An important stage of stellar formation where the star is still gaining (accreting) mass from the dust and gas around it and its core isn't hot enough to sustain fusion. The dust that surrounds all newly forming stars limits our ability to observe them; dust absorbs light across the entire electromagnetic spectrum, particularly high-energy wavelength, and re-emit it as infrared light. So, JWST is able see this light to reveal the protostars.

(Rabbit Hole: Learn more about dust and infrared astronomy here: basic, intermediate, and advanced.[1])

[1] Resources credited to the 2019 MIT Astronomy Walkthrough by Dhruva Karkada.

2. Located in Virgo, this stellar remnant was found to have a planetary system.
   (a) [1.5 pts] Name this object.
   (b) [2 pts] What type of object is it?
   (c) [2 pts] How many planet(s) have been discovered in this system?
   (d) [3 pts] Briefly (1-2 sentences) describe how the planets were discovered: (i) what type of data was collected, (ii) what telescope collected it, and (iii) how the planets affected the data.

Solution:

(a) Similar to the previous problem, there are only two objects that have a stellar remnant: PSR B1257+12 and WD 1856+534. In this case, our object is PSR B1257+12, as the latter is located in the constellation Draco.

(Rabbit Hole: Though the focus this year is not on stellar remnants, they are still a key phase of the stellar life cycle. Here are some resources introducing stellar remnants: basic and intermediate.)

(b) A (millisecond) pulsar! An introduction to pulsars—how they were discovered and what they are—can be found here and here.

(c) There are three confirmed planets.

(d) The Arecibo telescope measured the time of arrivals (TOAs) of the pulses from the pulsar. The gravitational perturbations by its orbiting planets resulted in the TOA to vary sinusoidally.

There aren't many articles that introduce this object, but as with any object, Wikipedia is always a good place to start. From there, we recommend looking through the references for any useful sources of more information. Enterprising competitors can read the paper announcing its discovery, Wolszczan, A. (1994), which is available for free through the WaybackMachine.

3. Image 3 is a plot of exoplanet related data.

   (a) [2 pts] What exoplanet detection method is shown by the plot?

   (b) [3 pts] Do you expect this exoplanet to have a circular or elliptical orbit? Explain your answer.

   (c) [2 pts] To three significant figures, what is the eccentricity of the exoplanet's orbit?

Solution:

(a) Radial-velocity or doppler spectroscopy. Primers on the radial-velocity method can be found here and here.

The topmost plot (a) shows a radial-velocity (RV) curve, which can be inferred from the y-axis units $\mathrm{km\,s^{-1}}$ and the x-axis units HJD (Heliocentric Julian Day) representing time in days.

(Rabbit Hole: What is time and how do astronomers ensure their measurements line up? It's more tricky than you'd expect! Do you look at the motion of the stars? Motion of the Sun? Astronomers have come up with a few different methods.)

The other two plots are derived from plot (a). Plot (b) is an observed-minus-calculated (O–C) plot showing the difference between the measured radial velocities (stars and boxes) and the fitted curve (solid line). (You may also see it called the fit error or the "residuals".) This plot is often included to highlight the accuracy of the fitted curve.

Plot (c) is a phase folded light curve, which "folds" points onto each other based on a defined period. This brings all of the measured data onto a single period of the velocity curve to further highlight the accuracy of the derived period. Finally, this plot also has a horizontal dotted line at the average radial velocity, or the systemic velocity, which is the velocity at which the system is moving with respect to our line of sight. In this case it is moving away from us at a rate of around $3.8\,\mathrm{km\,s^{-1}}$.

(b) It's in an elliptical orbit. If it was in a circular orbit, it would exhibit a sinusoidal radial-velocity curve, which is not the case.

To build intuition, play around with this radial-velocity simulator from the astronomy folk at UNL! Test out different eccentricities and inclinations. What does the curve look like when you set the eccentricity to 0? To 0.5? How does the inclination affect the curve?

(c) Based on how extreme the curve looks—compared to "standard" sinusoidal one—we can assume the eccentricity is relatively high; however, it is impractical, and pretty much impossible, to find the eccentricity from the curve itself. We expected you to lean on the information you collect on the DSOs and notice that HD 80606b is well known for being in a highly eccentric orbit. And indeed, this is a plot of HD 80606b which has an eccentricity of 0.923 to 0.934.

4. Images 4 and 5 show the temperature maps of WASP-121b, using a Mollweide projection, at pressures 5×10^3 Pa and 10^5 Pa, respectively.
   (a) [2 pts] Which image corresponds to a temperature map at a higher altitude? Explain your answer.
   (b) [2 pts] The temperature maps, particularly Image 5, suggest the existence of what type of phenomenon?
   (c) [3 pts] Considering your answer to part (a), explain why the patterns in the temperature maps differ.

Solution:

(a) Image 4 because pressure decreases with altitude and it is at a lower pressure. Many teams gave the right answer, but gave the wrong justification: "the temperatures in Image 4 are hotter, so it is higher up." This is not always true! Typically, the temperature in an atmosphere actually decreases as we move up in atmosphere. One reason for this is because the incident stellar energy (from the Sun or a host star) passes through the atmosphere and is absorbed by the Earth's or the planet's surface, heating it up. This heat is then transferred through the air by conduction, a slow process. Another reason is that, since pressure decreases as you move up, the air must expand and do work against its surrounding, which decreases its internal energy and therefore its temperature as well. (Recall the ideal gas law, *piv-nert!*)

In reality, there are instances where temperature does go up as you go up in altitude, called *thermal inversions*. The stratosphere here on Earth is an example of one.

(b) The patterns of flows and eddies in the temperature maps indicate weather patterns with storms and fronts.

(c) Comparing the temperature maps, we can visually see that the weather patterns are less prominent in Image 4. Specifically, the center of the temperature map corresponds to the point on the planet's surface pointing directly towards its star (known as the *substellar point*) explaining why the two quadrants in the center (dayside) are much hotter than the quadrants on the sides (nightside). This is due to the high stellar irradiation coupled with the short thermal timescale at that high altitude. (WASP-121b is also *tidally locked* which further contributes to the dayside–nightside thermal gradient.)

This question is heavily inspired by this ESA Hubble article and the associated paper.

5. LTT 9779b is an exoplanet discovered in 2020.
   (a) [3 pts] If the Moon orbits Earth at a distance of 3.85×10^8 m, how many times further does LTT 9779b orbit from its host star? Give your answer to the nearest whole number.
   (b) [2 pts] Image 6 shows all currently confirmed exoplanets and their method of discovery plotted against mass, radius, and orbital period. Why did the authors only include the "doppler" exoplanets in the upper plot?
   (c) [2 pts] The same image shows LTT 9779b sitting in a region devoid of other exoplanets. What is this region known as?
   (d) [3 pts] What is the prevailing theory for the lack of exoplanets in this region?

Solution:

(a) This question is a quick knowledge check with a calculation mixed in. Using the orbital radius from NASA's exoplanet database entry for LTT 9779b, we convert units and get

$$0.016\,79\,\mathrm{au} \times \frac{1.496 \times 10^{11}\,\mathrm{m}}{1\,\mathrm{au}} \times \frac{1}{3.85 \times 10^8\,\mathrm{m}} = \boxed{6.52}.$$

Rounding either up or down gives us 6 or 7.

LTT 9779b is super close to its host star! If the Sun was this close, it would have an angular diameter of 30°.

(Aside: It's a good idea to keep a document of common units and their conversions, particularly lengths—astronomical units, parsecs, and light-years—and masses—Solar, Earth, and Jupiter.)

(b) The upper plot shows the location of exoplanets on a period-mass plot whereas the lower plot shows them on a period-radius plot. By itself, doppler (spectroscopy) can't determine the radius of exoplanets, so it wouldn't make sense for the authors to include them in the lower one—since they don't have the data!

(c) This region is known as the (hot) Neptune desert.

(d) If an exoplanet is located close to their host star, they're subject to intense stellar radiation which over time will heat up the planet's atmosphere causing it to slowly leak out. This means that either the atmosphere will be completed eroded, leaving behind a rocky core or the exoplanet is large enough to hold onto its atmosphere (à la a hot Jupiter).

Learn more about Neptunian exoplanets and other planet types.

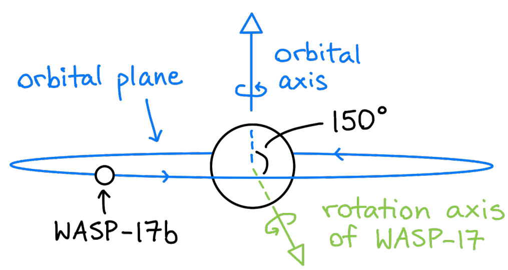
6. WASP-17b was the first exoplanet discovered to have a retrograde orbit.
   (a) [3 pts] This discovery was surprising because it was not predicted by planetary formation theory. Briefly (1-2 sentences) explain why astronomers expect planets to orbit in the same direction and their star's spin.
   (b) [3 pts] The sky-projected angle between the rotation axis of WASP-17 and the orbital axis of WASP-17b was measured to be around 150°. Draw a diagram showing this angle. Clearly denote the rotation and orbital axes by following the right-hand rule.
   (Hint: Draw the orbit of the exoplanet edge-on.)
   (c) [2 pts] What phenomenon was used to find this angle?

Solution:

(a) Young stars possess protoplanetary disks, both of which spin in the same direction. Since planets form from protoplanetary disks, from conservation of angular momentum, we'd expect planets to spin in the same direction as well.

Here are some resources about planetary formation: basic, intermediate, and advanced[2].

(b) This question is an exercise in visualizing the dynamics of a planetary system and communicating this through a diagram. The diagram should clearly mark out the orbital motion of WASP-17b and the rotation of WASP-17. Make sure to follow the right-hand rule when drawing rotation axes!



(c) Rossiter–McLaughlin effect.

This is a more advanced exoplanet topic; feel free to read more about it here.

[2]Ch. 15 of *Fundamental Planetary Science* (2019) by Jack J. Lissauer

7. Almost all of the objects in this year's deep-sky object list are related to exoplanets.
   (a) [3 pts] Identify the objects that satisfy at least one of the following conditions:
   - If the object is an exoplanet, it was first detected by transit.
   - If the object is a host star, at least one of its planets was first detected by transit.

   You may instead identify the complement: all objects not satisfying any of the conditions. If you do so, start your answer with "NOT: ".

   (b) [2 pts] From the objects not satisfying any of the conditions, find the median distance in parsecs. *(Hint: There are an odd number of these objects.)*

   (c) [2 pts] Image 7 displays what exoplanet detection method?

Solution:

(a) HD 80606, WASP-17b, WASP-121b, LTT 9779b, GJ 1214 b, K2-18b, TOI-270d, LHS 3844b, WD 1856+534, Kepler-62, AU Microscopii (or NOT: Orion Nebula, 30 Doradus, PSR B1257+12, 55 Cancri, Epsilon Eridani)

This is another knowledge check question. It's important to collect basic facts about all of the objects: what type of object it is; if it's an exoplanet, what detection method was used in its discovery; how far it away it is; how old it is; what constellation it is in; etc. Putting all this information into a summary document or a spreadsheet will speed things up during the exam!

(b) Out of the five objects, Orion Nebula is located 412 pc away.

(c) Direct imaging.

This video introduces this exoplanet detection method. This slide deck (PDF, PPT) goes into more depth by setting up other considerations with direct imaging.

Section D: Astrophysics

This section consists of calculations and a few derivations. Points are shown for each sub-question, for a total of 37 points. Numerical answers must be provided to 3 significant figures. Please show your work: no work, no points. Partial credit may be awarded for correct work.

1. **A Sumerian King.** You have been studying the main sequence star Gilgamesh, which you have determined is 2.5 parsecs away.
   (a) [3 pts] If you are observing Gilgamesh's motion for a 1-year period, what is its maximum angular displacement in the sky?
   (b) [3 pts] If Gilgamesh has an apparent bolometric magnitude of 2.5, what is its absolute magnitude?
   (c) [3 pts] What is the luminosity of Gilgamesh in solar luminosities?
   (d) [3 pts] If Gilgamesh has a radius of 0.76 solar radii, what is its effective temperature in Kelvin? (If you were unable to answer part (c), you may substitute it with a value of $0.7\,\mathrm{L}_\odot$.)
   (e) [3 pts] The black-body radiation curve emitted by Gilgamesh peaks at what wavelength in nm? (If you were unable to answer part (d), you may substitute it with a value of $4160\,\mathrm{K}$.)
   (f) [4 pts] You discover that Gilgamesh has an exoplanet, Enkidu! If Enkidu has an albedo of 0.4 and is orbiting at a distance of 1.4 au, what is the effective surface temperature of Enkidu in Kelvin? Is it in the habitable zone? (You may substitute the effective temperature of Gilgamesh with $4160\,\mathrm{K}$.)

Solution:

(a) This question tests your understanding of parallax. Parallax is defined by the angle subtended over a baseline of 1 au. But since we are observing Gilgamesh over the course of an entire year, the Earth will have a maximum separation of 2 au. So we need to double the angle from the usual parallax equation.

$$p = \frac{1}{d} = \frac{1}{2.5\,\mathrm{pc}} = 0.4'' \qquad \Longrightarrow \qquad 2p = \boxed{0.8''}$$

where p is the parallax angle and d is the distance.

Partial credit of 1 point was given for $0.4''$.

(b) Solve for M in the distance modulus equation to find

$$2.5 - M = 5\log_{10}\left[\frac{2.5\,\mathrm{pc}}{10\,\mathrm{pc}}\right] \qquad \Longrightarrow \qquad M = \boxed{+5.51.}$$

See §5.3 of *A Student's Guide to the Mathematics of Astronomy* (2013) written by Daniel Fleisch and Julia Kregenow (hereafter referred to as GMA).

(c) To find the luminosity of Gilgamesh, we need to convert absolute magnitude into luminosity. This can be done by understanding magnitude represents a ratio of luminosity (or brightness) where 5 magnitude steps correspond to 100 times luminosity (or brightness) (§5.3 in GMA describes this in more detail).

The luminosity–magnitude relation is

$$M - M_\odot = -2.5\log_{10}\left[\frac{L}{\mathrm{L}_\odot}\right],$$

where $\mathrm{L}_\odot$ represents 1 solar luminosity and $M_\odot$ is the absolute magnitude of the Sun. In this case, since we are looking at the bolometric magnitude of Gilgamesh, we should use the bolometric magnitude of the Sun, which is +4.74. Teams that used the visual absolute magnitude $M_V = +4.83$ were not penalized.

Plugging in and solving, we find

$$5.51 - 4.74 = -2.5\log_{10}\left[\frac{L}{\mathrm{L}_\odot}\right] \implies L = \boxed{0.492\,\mathrm{L}_\odot.}$$

(d) For this question, we'll use the Stefan–Boltzmann law (§3.2.2 in GMA) relating luminosity, radius, and (surface) temperature:

$$\frac{L}{\mathrm{L}_\odot} = \left(\frac{R}{\mathrm{R}_\odot}\right)^2\left(\frac{T}{\mathrm{T}_\odot}\right)^4,$$

where $\mathrm{R}_\odot$ represents 1 solar radius and $\mathrm{T}_\odot = 5772\,\mathrm{K}$ represents the surface temperature of the sun. Plugging in the values we know, we calculate

$$\frac{0.492\,\mathrm{L}_\odot}{\mathrm{L}_\odot} = \left(\frac{0.76\,\mathrm{R}_\odot}{\mathrm{R}_\odot}\right)^2\left(\frac{T}{5772\,\mathrm{K}}\right)^4 \implies T = \boxed{5550\,\mathrm{K}.}$$

(e) Using Wien's law (see §3.2 of GMA), we get:

$$\lambda_{\mathrm{peak}} = \frac{b}{T_{\mathrm{eff}}} = \frac{2.898\times10^6\,\mathrm{nm\,K}}{5550\,\mathrm{K}} = \boxed{521\,\mathrm{nm}.}$$

(f) The surface temperature, also known as the equilibrium temperature, of a planet depends on a balance of energy absorbed from its star and the energy emitted from the planet. In its most simple form, it written as

$$T_{\mathrm{p}} = T_\star\left(\frac{R_\star}{2d}\right)^{1/2}(1-A)^{1/4},$$

where $T_\star$ and $R_\star$ are the temperature and radius of the host star (Why do we use $\star$ instead of $\odot$?), d is the distance between the planet and the star, and A is the albedo. An explanation of the formula can be found here.

Plugging in, we get

$$T_{\mathrm{p}} = 5550\,\mathrm{K}\left(\frac{0.76\,\mathrm{R}_\odot}{1.4\,\mathrm{au}}\right)^{1/2}(1-0.4)^{1/4} = \boxed{173\,\mathrm{K}.}$$

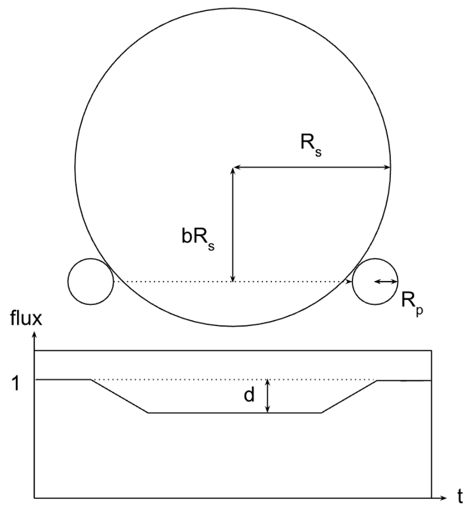
Make sure to convert $R_\star$ and d to the same units!

(Rabbit Hole: This model of the surface temperature of a planet is an extremely simple one. (What assumptions does it make?) A more advanced resource can be found here[3]. §26.1 derives the equation we just used and §26.2 adds in the "greenhouse effect" by including a one-layer atmosphere.)

[3] We can't figure out what textbook this is taken from. So we apologize in advance if the link breaks.

2. **Exoplanet Detection!** Let's take a closer look at the two most common exoplanet detection methods. Transit detection has confirmed over 4,000 exoplanets so far. For close to edge-on orbits ($i \sim 90°$), as a planet orbits its host star, it will cause a dip in the observed stellar flux as it transits in front of the star.

   For the following questions, assume a circular orbit. The diagram below shows a transiting exoplanet and the system's associated light curve. R_s and R_p are the radii of the star and planet, respectively. For an edge-on orbit, the planet transits along the equator of the host star. For i near $90°$, it transits along a chord a distance bR_s away from the star's center, where $0 \leqslant b \leqslant 1$ is a constant termed the *impact parameter*.



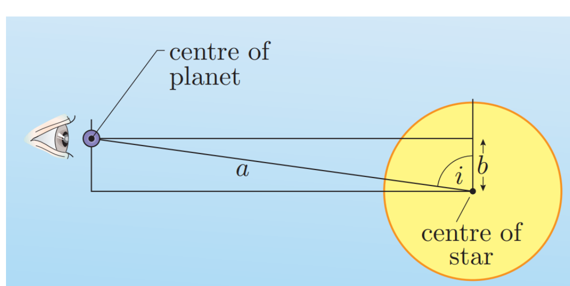
   (a) [4 pts] What is the maximum and minimum inclination for the transit to be visible? Express your answer in terms of the parameters in the image above and the semi-major axis of the orbit, a.

   (b) [3 pts] What is the maximum possible dip in flux observed, d, as a fraction of the total stellar flux? Express your answer in terms of R_p and R_s.

Solution: Exoplanets are one of the key focus areas for this year. Vox has an introduction video to exoplanets and their various detection methods.

These two questions go over some of the key derivations for the transit method. Here is a video lecture discussing it. Also, *Transiting Exoplanets* (2010) by Carole A. Haswell (hereafter TE) is a wonderful introduction to transits and covers these derivations in more depth.

(a) The following diagram (Figure 3.2 in TE) shows the transit geometry. Note that with our definition of b, the diagram should have bR_s in place of b.



Using trigonometric relations, we find that $a\cos(i) = bR_s$. Since $0 \leqslant b \leqslant 1$, we can substitute in this inequality to find

$$\boxed{i \geqslant \cos^{-1}\left(\frac{R_s}{a}\right).}$$

This means that if i gets too small, the orbit of the exoplanet will be too far up to move in front of the star to make a transit.

The inequality $i \geqslant \cos^{-1}((R_s + R_p)/a)$ was also accepted, because it takes into account the radius of the planet. We accepted both responses since the radius of the star is typically much greater than the radius of the planet ($R_s \gg R_p$).

§3.1.3 of TE goes over impact parameters in more depth.

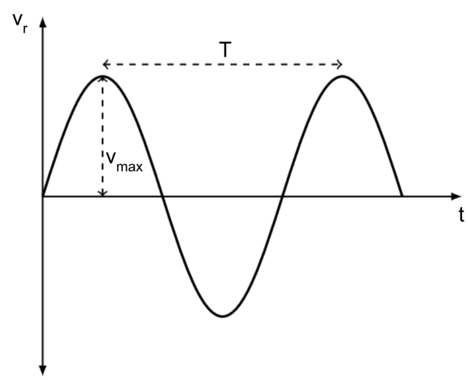
(b) Since we are viewing the system from extremely far away, the exoplanet and the star look like disks of radius R_p and R_s, respectively. The maximum decreases in flux received from the star occurs when the planet's "disk" is fully inside of the stellar "disk". Then, the percent *decrease* in the stellar disk we see is exactly

$$\boxed{d = \left(\frac{R_p}{R_s}\right)^2.}$$

Partial credit of 1 point was given to teams who gave the expression for $1 - d = (R_s^2 - R_p^2)/R_s^2$.

Read §1.4 in TE for more information.

Another popular form of exoplanet detection is the radial velocity method, which utilizes the Doppler "wobble" emerging from the gravitational force exerted on the host star. This Doppler shift can be measured to create a radial velocity curve of the host star over time. A radial velocity curve is shown below with a maximum observed speed of $v_{\max}$ and period T. For the following questions, assume the planet's motion follows an edge-on, circular orbit.



(c) [4 pts] Suppose that we have been able to determine the mass of the host star M_s. What is the orbital speed of the planet v_p? Give your answer in terms of observed quantities from the radial velocity curve (T, $v_{\max}$), as well as fundamental constants.

(d) [4 pts] What is the mass of the planet, M_p, in terms of the host star mass M_s, its radial velocity amplitude $v_{\max}$, and the inferred planet speed v_p?

(e) [3 pts] How does your expression for M_p change when the orbit cannot be assumed to be edge-on ($i \neq 90°$)?

Solution:

(c) Since we have the orbit period T and the mass of the star M_s, we can use Kepler's third law to get the semi-major axis of the orbit a:

$$a^3 = \frac{GM_s}{4\pi^2}T^2 \quad \Longrightarrow \quad a = \left(\frac{GM_s}{4\pi^2}T^2\right)^{1/3},$$

where G is the gravitational constant.

Since we are assuming a circular orbit, the semi-major axis of the orbit is the same as the radius. We can use the circular orbit velocity formula and substitute in our semi-major axis to get

$$v_p = \sqrt{\frac{GM_s}{a}} = \boxed{\left(\frac{2\pi GM_s}{T}\right)^{1/3}.}$$

§2.3 of SMA goes over Kepler's laws.

(d) Because the planet and host star orbit around their center of mass, $M_p v_p = M_s v_s$, where v_s is the true velocity of the star, which equals $v_{\max}$ in the case of an edge-on orbit. So we can write:

$$\boxed{M_p = \frac{M_s v_{\max}}{v_p}.}$$

Partial credit of 3 points was given for $M_p = (M_s v_s)/v_p$.

(e) If the orbit is not edge-on, $v_{\max} = v_s \sin(i)$, as $v_{\max}$ is the radial component of the star's true velocity. So we get:

$$\boxed{M_p = \frac{M_s v_{\max}}{v_p \sin(i)}.}$$

§3.4 of SMA goes over radial-velocity plots and discusses the effect of inclination (see Figure 3.19).